Chapter 1: Problem Solutions

Review of Signals and Systems

Signals

■ Problem 1.1

a) $x[n] = -0.5\,\delta[n+1] + \delta[n] + 0.5\,\delta[n-1] + \delta[n-2] - 0.8\,\delta[n-3]$

b) $x[n] = -0.5\,\delta[n+5] + \delta[n+4] + 0.5\,\delta[n+3] + \delta[n+2] - 0.8\,\delta[n+1]$

■ Problem 1.2

a) $I = e^{-1}$

b) $I = e^{-1}$

c) $I = 0$ since the interval of integration does not include the point $t = -1$, where the impulse is centered.

d) $I = 1$

e) $I = \cos^2(0.1\,\pi)$

f) $I = e$

g) let $\lambda = -\frac{1}{2}\,t$, then $t = -2\,\lambda$ and $dt = -2\,d\lambda$. Substitute in the integral to obtain

$$I = \int_{+\infty}^{-\infty} (-2\,\lambda)^2\,\delta\left(\lambda + \tfrac{1}{2}\right)\,(-2\,d\lambda) =$$

$$= \int_{-\infty}^{+\infty} 8\,\lambda^2\,\delta\left(\lambda + \tfrac{1}{2}\right)\,d\lambda = 2$$

h) let $\lambda = 3\,t$, then $t = \lambda / 3$ and $dt = \left(\frac{1}{3}\right)\,d\lambda$. Then the integral becomes

$$I = \int_{-\infty}^{+\infty} e^{\lambda/3}\,\delta\,(\lambda - 1)\,\left(\tfrac{1}{3}\right)\,d\lambda = \frac{e^{1/3}}{3}$$

■ Problem 1.3

Amplitude $A = 2$,

Period $T_0 = 0.01\ \text{sec}$, then frequency $F_0 = \frac{1}{T_0} = 100\ \text{Hz} = 0.1\ \text{kHz}$

Phase $\alpha = 0$

Then the signal can be written as $x(t) = 2\cos(200\pi t)$.

■ Problem 1.4

a) Frequency $F_0 = 1/T_0 = 1 \big/ \left(3 \times 10^{-3}\right) = \frac{1}{3} \times 10^3\ \text{Hz}$. Then

$$x(t) = 2.5\cos\left(\frac{2}{3}\,1000\,\pi t + 15^0\right)$$

b) Digital Frequency $\omega_0 = 2\pi F_0 / F_s = 2\pi/6 = \pi/3\ \text{rad}$. Therefore the sampled sinusoid becomes

$$x[n] = 2.5\cos\left(\frac{\pi}{3}\,n + 15^0\right)$$

■ Problem 1.5

All sinusoids are distinct. In continuous time there is no ambiguity between

frequency and signal.

■ Problem 1.6

First bring all frequencies within the interval $-\pi$ to π. This yields

$$x_2[n] = 2\cos(1.5\pi n - 0.1\pi - 2\pi n) = 2\cos(-0.5\pi n - 0.1\pi)$$
$$= 2\cos(0.5\pi n + 0.1\pi)$$

and also

$$x_3[n] = 2\cos(1.5\pi n + 0.1\pi - 2\pi n) = 2\cos(-0.5\pi n + 0.1\pi)$$
$$= 2\cos(0.5\pi n - 0.1\pi)$$

Therefore we can see that $x_1[n] = x_2[n]$ and $x_3[n] = x_4[n]$.

A different way of solving this problem is graphically. The frequency

plots for all four signals are shown next. The key point is to understand

that in discrete time, all frequencies within the interval $-\pi,\ \pi$ yield complex Exponentials with distinct values:

$x_1[n]$ Frequency Representation

$x_2[n]$ Frequency Representation

$x_3[n]$ Frequency Representation

$x_4[n]$ Frequency Representation

Again you see that $x_1[n]$ and $x_2[n]$ have the same representation, and the same for $x_3[n]$ and $x_4[n]$.

■ Problem 1.7

The sinusoid $x[n] = 3\cos(1.9\pi n + 0.2\pi)$ has the same samples, since

$$3\cos(0.1\pi n - 0.2\pi) = 3\cos(-0.1\pi n + 2\pi n + 0.2\pi)$$